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Addendum to "Impressed sources and fields in the volume-integral-equation formulation of electromagnetic scattering by a finite object: A tutorial" [J. Quant. Spectrosc. Radiat. Transfer 214 (2018) 158–167]

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ABSTRACT

Our recent tutorial referred to in the title has summarized a general theoretical formalism of electromagnetic scattering by an arbitrary finite object in the presence of arbitrarily distributed impressed currents. This addendum builds on the tutorial to provide a streamlined discussion of specific far-field limits and the corresponding reciprocity relations by introducing appropriate far-field operators and linear maps and deriving the reciprocity relations through the pseudo adjoint of these maps. We thereby extend the compact operator calculus used previously to consider the fields and sources near or inside the scattering object.

1. Introduction

In a recent tutorial [1], we have outlined a general and self-consistent theoretical formalism describing frequency-domain electromagnetic scattering by an arbitrary finite object in the presence of arbitrarily distributed impressed currents. Sections 9–11 of Ref. [1] dealt with "far-field" limits, i.e., with scenarios wherein the source and/or observation points are far from the scatterer. The corresponding derivations are straightforward and successful in proving fundamental reciprocity relations. However, they are somewhat cumbersome and do not employ the operator calculus invoked in Sections 7, 8, and 12 to streamline the derivations for fields and sources near or inside the scattering object. The goal of this addendum is to remedy these deficiencies by introducing far-field operators and linear maps and deriving the reciprocity relations through the pseudo adjoint of these maps. Not being constrained by the scope of the original tutorial, we pay more attention to mathematical rigor.

2. Notation and function spaces

For the sake of consistency, we keep the notation of the original tutorial [1]. In particular, bold letters denote vectors ($\mathbf{a}$, $\mathbf{A}$); bold letters with a caret denote unit vectors ($\hat{\mathbf{a}}$); Italic capital letters with a double-headed arrow denote dyadics ($\overleftrightarrow{A}$) or, depending on the context, their matrix representations; Italic letters denote various variables or elements of a function space (a, A); and Italic capital letters with a caret denote linear operators or maps acting between these spaces ($\hat{A}$). We reserve the term "operator" for maps between a space and itself, and use handwritten capital letters to denote function spaces themselves ($\mathcal{A}$).

First, let us define the space of (near) fields $\mathcal{H}_\mathrm{n} \stackrel{\mathrm{def}}{=} L^2(\mathbb{R}^3)^3$ such that each Cartesian component of a field is square-integrable. This space was implicitly assumed in Ref. [1] for electric fields and sources when the latter have finite support, i.e., are not zero only in a finite volume. Moreover, this space naturally corresponds to the finiteness of the energy of the electromagnetic field [2]. Second, we define the space of square-integrable transverse far-fields defined on a unit sphere S^2: $\mathcal{H}_\mathrm{f} \stackrel{\mathrm{def}}{=} \{E | E \in L^2(S^2)^3, \forall \hat{\mathbf{n}} \in S^2 : \mathbf{E}(\hat{\mathbf{n}}) \perp \hat{\mathbf{n}}\}$. Owing to the orthogonality condition, the fields in $\mathcal{H}_\mathrm{f}$ are effectively two-dimensional,

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i.e., $\mathcal{H}_\mathrm{f}$ is isomorphic (equivalent) to $\mathcal{H}_\mathrm{f2} \stackrel{\mathrm{def}}{=} L^2(S^2)^2$. The corresponding isomorphism $\hat{Q}: \mathcal{H}_\mathrm{f} \to \mathcal{H}_\mathrm{f2}$ is local with respect to $\hat{\mathbf{n}}$, i.e.,

$$\mathbf{E}_{(2)}(\hat{\mathbf{n}}) = \overleftrightarrow{Q}(\hat{\mathbf{n}})\mathbf{E}_{(3)}(\hat{\mathbf{n}}), \tag{1}$$

where parenthesized superscripts denote the corresponding dimensionality of the vectors or dyadics (if necessary) and $\overleftrightarrow{Q}(\hat{\mathbf{n}})$ is the 2×3 matrix defining row-wise the orthonormal basis on a sphere (thereby stretching the dyadic notation). We do not specify a particular basis (e.g., the standard spherical basis vectors $\hat{\mathbf{e}}_\theta$ and $\hat{\mathbf{e}}_\varphi$) and only require it to be real, thereby implying that $\overleftrightarrow{Q}$ is row-orthogonal, i.e.,

$$\overleftrightarrow{Q}(\hat{\mathbf{n}}) \cdot \overleftrightarrow{Q}^\mathrm{T}(\hat{\mathbf{n}}) = \overleftrightarrow{I}_{(2)}, \quad \overleftrightarrow{Q}^\mathrm{T}(\hat{\mathbf{n}}) \cdot \overleftrightarrow{Q}(\hat{\mathbf{n}}) = \overleftrightarrow{I} - \hat{\mathbf{n}} \otimes \hat{\mathbf{n}}, \tag{2}$$

where $\otimes$ denotes the dyadic product, the superscript "T" denotes the standard transposition of a dyadic, and $\overleftrightarrow{I}$ is the identity dyadic. The second equality follows from uniqueness of the projector matrix on the one-dimensional subspace parallel to $\hat{\mathbf{n}}$. Furthermore, $\overleftrightarrow{I} - \hat{\mathbf{n}} \otimes \hat{\mathbf{n}}$ is equivalent to $\overleftrightarrow{I}$ when acting on a transverse field, hence $\overleftrightarrow{Q}^\mathrm{T}(\hat{\mathbf{n}})$ defines the inverse isomorphism $\hat{Q}^{-1}: \mathcal{H}_\mathrm{f2} \to \mathcal{H}_\mathrm{f}$. In the following, we will mostly deal with $\mathcal{H}_\mathrm{f}$ to be independent of $\overleftrightarrow{Q}(\hat{\mathbf{n}})$, but we have to consider $\mathcal{H}_\mathrm{f2}$ in discussing the scattering matrices.

3. Main results

The main entity in considering far-field quantities is the limiting linear map defined as

$$\hat{F}: \mathcal{H}_\mathrm{n} \to \mathcal{H}_\mathrm{f}, \quad \left(\hat{F}E\right)(\hat{\mathbf{n}}) \stackrel{\mathrm{def}}{=} \lim_{r\to\infty} r \exp(-\mathrm{i}k_1 r)(\overleftrightarrow{I} - \hat{\mathbf{n}} \otimes \hat{\mathbf{n}}) \cdot \mathbf{E}(r\hat{\mathbf{n}}), \tag{3}$$

where the limit exists due to square integrability in $\mathbb{R}^3$, while the projector $(\overleftrightarrow{I} - \hat{\mathbf{n}} \otimes \hat{\mathbf{n}})$ is added for convenience to limit the range of the map to $\mathcal{H}_\mathrm{f}$. We neither use nor analyze $\hat{F}$ separately. Instead we always combine it with a Green's operator (explicitly or implicitly), in which case the above projector becomes redundant. We start with

$$\hat{G}_\mathrm{f}: \mathcal{H}_\mathrm{n} \to \mathcal{H}_\mathrm{f}, \quad \hat{G}_\mathrm{f} \stackrel{\mathrm{def}}{=} \hat{F}\hat{G}, \quad \left(\hat{G}_\mathrm{f} j\right)(\hat{\mathbf{n}}) = \int_{\mathbb{R}^3} \mathrm{d}^3\mathbf{r}\, \overleftrightarrow{G}_\mathrm{f}(\hat{\mathbf{n}}, \mathbf{r}) \cdot \mathbf{j}(\mathbf{r}), \tag{4}$$

$$\overleftrightarrow{G}_\mathrm{f}(\hat{\mathbf{n}}, \mathbf{r}) = \frac{1}{4\pi} \exp(-\mathrm{i}k_1 \hat{\mathbf{n}} \cdot \mathbf{r})(\overleftrightarrow{I} - \hat{\mathbf{n}} \otimes \hat{\mathbf{n}}), \tag{5}$$

where $\hat{G}$ is the free-space Green's operator given by Eq. (18) of Ref. [1] and Eq. (5) is equivalent to Eq. (68) of Ref. [1]. Typically – e.g., Eq. (12) of Ref. [1] – the forcing function j has finite support V, implying $j \in L^2(V)^3 \subset \mathcal{H}_\mathrm{n}$. The scattered far-field is expressed as

$$E_\mathrm{f}^\mathrm{sca} \stackrel{\mathrm{def}}{=} \hat{F}E^\mathrm{sca} = \hat{G}_\mathrm{f}\hat{U}E = \hat{G}_\mathrm{f}\hat{T}E^\mathrm{S}, \quad E^\mathrm{sca} \stackrel{\mathrm{def}}{=} E - E^\mathrm{S}, \tag{6}$$

which follows from Eqs. (22), (38), (39), and (69) of Ref. [1]. Here E, E^S, and E^sca denote the total, incident (due to impressed sources), and scattered fields, respectively. The operator $\hat{U}$ defines the scatterer and the transition operator $\hat{T}$ fully describes its electromagnetic response as defined by Eqs. (13), (25), and (35) of Ref. [1].

Let us further define the *pseudo adjoint* [2] (or transpose [3]) of $\hat{G}_\mathrm{f}$, denoted as $\hat{G}_\mathrm{f}^\mathrm{t}: \mathcal{H}_\mathrm{f} \to \mathcal{H}_\mathrm{n}$. It is done analogously to the Hermitian adjoint,

$$\left\langle j, \hat{G}_\mathrm{f}^\mathrm{t} E\right\rangle_{\mathcal{H}_\mathrm{n}}^\mathrm{p} \stackrel{\mathrm{def}}{=} \left\langle \hat{G}_\mathrm{f} j, E\right\rangle_{\mathcal{H}_\mathrm{f}}^\mathrm{p}, \quad \forall E \in \mathcal{H}_\mathrm{f}, \forall j \in \mathcal{H}_\mathrm{n}, \tag{7}$$

but using the pseudo inner products

$$\langle a, b\rangle_{\mathcal{H}_\mathrm{f}}^\mathrm{p} \stackrel{\mathrm{def}}{=} \int_{S^2} \mathrm{d}^2\hat{\mathbf{n}}\, \mathbf{a}(\hat{\mathbf{n}}) \cdot \mathbf{b}(\hat{\mathbf{n}}), \quad \langle a, b\rangle_{\mathcal{H}_\mathrm{n}}^\mathrm{p} \stackrel{\mathrm{def}}{=} \int_{\mathbb{R}^3} \mathrm{d}^3\mathbf{r}\, \mathbf{a}(\mathbf{r}) \cdot \mathbf{b}(\mathbf{r}) \tag{8}$$

instead of the standard L^2 inner products, i.e., differing by complex conjugation of the second argument. Mathematically, the pseudo inner product is *bilinear*, i.e., linear in both arguments, while the standard inner product is *sesquilinear*, i.e., linear in the first argument but antilinear (conjugate-linear) in the second one. There is no clear consensus in the literature on the proper symbol for pseudo adjoint, but we use the superscript "t" to distinguish it from the dyadic transpose, although the two concepts are closely related.

The pseudo adjoint was defined (less rigorously) for operators from $\mathcal{H}_\mathrm{f}$ to $\mathcal{H}_\mathrm{f}$ by Eq. (45) of Ref. [1], together with the notion of *pseudo self-adjointness*, i.e., the operator being equal to its pseudo adjoint. In particular, we use in the following the fact that the operators $\hat{G}$, $\hat{U}$, and $\hat{T}$, as well as the source Green's operator $\hat{G}_\mathrm{S}$ are pseudo self-adjoint for any reciprocal medium (Eqs. (32), (57), and (58) of Ref. [1]).

Taking pseudo adjoint has many properties of the matrix transposition or taking adjoint [2], most importantly

$$\left(\hat{A}\hat{B}\right)^\mathrm{t} = \hat{B}^\mathrm{t}\hat{A}^\mathrm{t}. \tag{9}$$

While any isomorphism conserves the inner product, Eq. (2) implies that $\hat{Q}$ also conserves the pseudo inner product, i.e.,

$$\left\langle \hat{Q}a, \hat{Q}b\right\rangle_{\mathcal{H}_\mathrm{f2}}^\mathrm{p} = \langle a, b\rangle_{\mathcal{H}_\mathrm{f}}^\mathrm{p}. \tag{10}$$

Thus in Eqs. (7) and (8), $\mathcal{H}_\mathrm{f}$ can be effectively replaced by $\mathcal{H}_\mathrm{f2}$ and

$$\hat{Q}^\mathrm{t} = \hat{Q}^{-1} \Rightarrow \left(\hat{Q}\hat{G}_\mathrm{f}\right)^\mathrm{t} = \hat{G}_\mathrm{f}^\mathrm{t}\hat{Q}^{-1}. \tag{11}$$

Alternatively, $\hat{G}_\mathrm{f}^\mathrm{t}$ can be expressed through the integral kernel

$$\left(\hat{G}_\mathrm{f}^\mathrm{t} E\right)(\mathbf{r}) = \int_{S^2} \mathrm{d}^2\hat{\mathbf{n}}\, \overleftrightarrow{G}_\mathrm{f}^\mathrm{t}(\mathbf{r}, \hat{\mathbf{n}}) \cdot \mathbf{E}(\hat{\mathbf{n}}), \tag{12}$$

which together with Eqs. (4) and (7) implies

$$\overleftrightarrow{G}_\mathrm{f}^\mathrm{t}(\mathbf{r}, \hat{\mathbf{n}}) = \left[\overleftrightarrow{G}_\mathrm{f}(\hat{\mathbf{n}}, \mathbf{r})\right]^\mathrm{T}. \tag{13}$$

The dyadic $\overleftrightarrow{G}_\mathrm{f}^\mathrm{t}$ can also be thought of as a free-space field from a distant source:

$$\overleftrightarrow{G}_\mathrm{f}^\mathrm{t}(\mathbf{r}, \hat{\mathbf{n}}) = \lim_{r'\to\infty} r' \exp\left(-\mathrm{i}k_1 r'\right)\overleftrightarrow{G}^\mathrm{t}(\mathbf{r}, r'\hat{\mathbf{n}}), \tag{14}$$

where the last superscript "t" can in principle be omitted owing to the pseudo self-adjointness of $\hat{G}$ and the invariance of $\overleftrightarrow{G}$ with respect to argument interchange. But the entire analysis is valid for an arbitrary background reciprocal medium, e.g., a semi-infinite plane substrate. Then $\hat{G}$ is still pseudo self-adjoint, but the arguments of $\overleftrightarrow{G}$ can be interchanged only when combined with transposition.

The linear map $\hat{G}_\mathrm{f}^\mathrm{t}$ constructs the field due to a distribution of distant sources and is related to the vector Herglotz wave function (Eq. (6.94) of Ref. [4]). In particular, the incident (source) plane wave with an amplitude $\mathbf{E}_0^\mathrm{inc}$ propagating along $\hat{\mathbf{n}}^\mathrm{inc}$ is given by

$$\mathbf{E}^\mathrm{S}(\mathbf{r}) \stackrel{\mathrm{def}}{=} \mathbf{E}_0^\mathrm{inc} \exp\left(\mathrm{i}k_1 \hat{\mathbf{n}}^\mathrm{inc} \cdot \mathbf{r}\right), \tag{15}$$

which can be transformed using Eqs. (5) and (13) into

$$E^\mathrm{S} = 4\pi \hat{G}_\mathrm{f}^\mathrm{t}\hat{H}E^\mathrm{inc}, \quad \mathbf{E}^\mathrm{inc}(\hat{\mathbf{n}}) = \mathbf{E}_0^\mathrm{inc}\delta(\hat{\mathbf{n}} - \hat{\mathbf{n}}^\mathrm{inc}), \tag{16}$$

where $\delta(\hat{\mathbf{n}})$ is the solid-angle delta function, while the linear operator $\hat{H}: \mathcal{H}_\mathrm{f} \to \mathcal{H}_\mathrm{f}$ inverts the direction of $\hat{\mathbf{n}}$ and can be represented as an integral operator with the kernel

$$\overleftrightarrow{H}(\hat{\mathbf{n}}, \hat{\mathbf{n}}') = \overleftrightarrow{I}\, \delta(\hat{\mathbf{n}} + \hat{\mathbf{n}}'). \tag{17}$$

Obviously, $\hat{H}^{\mathrm{t}} = \hat{H}^{-1} = \hat{H}$. Eq. (16) can also be obtained by explicitly moving the source to infinity (with a linear scaling of its amplitude) according to Eq. (14), as was done in Ref. [1]. Strictly speaking, neither E^{inc} nor E^{S} in Eq. (16) are square-integrable due to the delta function, i.e., $E^{\mathrm{inc}} \notin \mathcal{H}_{\mathrm{f}}$ and $E^{\mathrm{S}} \notin \mathcal{H}_{\mathrm{n}}$. However, they can be defined as the limits of sequences from these spaces (i.e., as generalized functions), and the central Eq. (7) remains valid since both pseudo inner products are well defined, albeit potentially unbounded if at least one of the arguments is square-integrable.

The far-field scattering linear operator $\hat{A} : \mathcal{H}_{\mathrm{f}} \to \mathcal{H}_{\mathrm{f}}$ with the dyadic kernel $\overleftrightarrow{A}(\hat{\mathbf{n}}^{\mathrm{sca}}, \hat{\mathbf{n}}^{\mathrm{inc}})$ is defined by Eq. (69) of Ref. [1] or, equivalently, as

$$\hat{A}E^{\mathrm{inc}} \overset{\mathrm{def}}{=} E_{\mathrm{f}}^{\mathrm{sca}} \tag{18}$$

(cf. Eq. (6.98) of Ref. [4]), which together with Eqs. (6) and (16) implies

$$\hat{A} = 4\pi \hat{G}_{\mathrm{f}} \hat{T} \hat{G}_{\mathrm{f}}^{\mathrm{t}} \hat{H} \Rightarrow \hat{A}^{\mathrm{t}} = \hat{H}\hat{A}\hat{H}, \tag{19}$$

since $\hat{T}$ is pseudo self-adjoint. The last part of Eq. (19) is exactly the far-field reciprocity relation:

$$\overleftrightarrow{A}\left(\hat{\mathbf{n}}^{\mathrm{sca}}, \hat{\mathbf{n}}^{\mathrm{inc}}\right) = \left[\overleftrightarrow{A}\left(-\hat{\mathbf{n}}^{\mathrm{inc}}, -\hat{\mathbf{n}}^{\mathrm{sca}}\right)\right]^{\mathrm{T}}. \tag{20}$$

The widely used amplitude scattering matrix [5] is the following 2×2 dyadic $\overleftrightarrow{A}_{(2)}$ corresponding to the operator $\hat{A}_{(2)} : \mathcal{H}_{\mathrm{f2}} \to \mathcal{H}_{\mathrm{f2}}$:

$$\overleftrightarrow{A}_{(2)}\left(\hat{\mathbf{n}}^{\mathrm{sca}}, \hat{\mathbf{n}}^{\mathrm{inc}}\right) \overset{\mathrm{def}}{=} \overleftrightarrow{Q}\left(\hat{\mathbf{n}}^{\mathrm{sca}}\right)\overleftrightarrow{A}\left(\hat{\mathbf{n}}^{\mathrm{sca}}, \hat{\mathbf{n}}^{\mathrm{inc}}\right)\overleftrightarrow{Q}^{\mathrm{T}}\left(\hat{\mathbf{n}}^{\mathrm{inc}}\right). \tag{21}$$

Substituting Eq. (21) into Eq. (20) we obtain the modified reciprocity relation

$$\overleftrightarrow{A}_{(2)}\left(\hat{\mathbf{n}}^{\mathrm{sca}}, \hat{\mathbf{n}}^{\mathrm{inc}}\right) = \left[\overleftrightarrow{R}\left(\hat{\mathbf{n}}^{\mathrm{inc}}\right)\overleftrightarrow{A}_{(2)}\left(-\hat{\mathbf{n}}^{\mathrm{inc}}, -\hat{\mathbf{n}}^{\mathrm{sca}}\right)\overleftrightarrow{R}^{\mathrm{T}}\left(\hat{\mathbf{n}}^{\mathrm{sca}}\right)\right]^{\mathrm{T}}, \tag{22}$$

where we used Eq. (2) and

$$\overleftrightarrow{A}\left(\hat{\mathbf{n}}^{\mathrm{sca}}, \hat{\mathbf{n}}^{\mathrm{inc}}\right) \cdot \hat{\mathbf{n}}^{\mathrm{inc}} = \hat{\mathbf{n}}^{\mathrm{sca}} \cdot \overleftrightarrow{A}\left(\hat{\mathbf{n}}^{\mathrm{sca}}, \hat{\mathbf{n}}^{\mathrm{inc}}\right) = 0 \tag{23}$$

(by definition of $\mathcal{H}_{\mathrm{f}}$). The 2×2 dyadic

$$\overleftrightarrow{R}\left(\hat{\mathbf{n}}\right) \overset{\mathrm{def}}{=} \overleftrightarrow{Q}\left(\hat{\mathbf{n}}\right)\overleftrightarrow{Q}^{\mathrm{T}}\left(-\hat{\mathbf{n}}\right) \tag{24}$$

takes a simple form in many natural bases. In particular, for a standard spherical basis $\overleftrightarrow{R}$ is a diagonal matrix with elements $+1$ and -1, then its effect in Eq. (22) amounts to inverting the sign of the off-diagonal elements (cf. Eq. (5.31) of Ref. [5]).

Analogously, the generalized scattering linear map $\hat{S} : \mathcal{H}_{\mathrm{f}} \to \mathcal{H}_{\mathrm{n}}$ with the kernel $\overleftrightarrow{S}(\mathbf{r}, \hat{\mathbf{n}})$ is given by

$$\hat{S}E^{\mathrm{inc}} \overset{\mathrm{def}}{=} E^{\mathrm{sca}} \Leftrightarrow \hat{A} = \hat{F}\hat{S}, \tag{25}$$

cf. Eq. (73) of Ref. [1]. Combining this definition with Eq. (18), we obtain

$$\hat{S} = 4\pi \hat{G}\hat{T}\hat{G}_{\mathrm{f}}^{\mathrm{t}}\hat{H} \Rightarrow \left(\hat{S}\hat{H}\right)^{\mathrm{t}} = 4\pi \hat{G}_{\mathrm{f}}\hat{T}\hat{G} = 4\pi\left(\hat{G}_{\mathrm{Sf}} - \hat{G}_{\mathrm{f}}\right), \tag{26}$$

$$\hat{G}_{\mathrm{Sf}} \overset{\mathrm{def}}{=} \hat{F}\hat{G}_{\mathrm{S}} = \hat{G}_{\mathrm{f}} - \hat{G}_{\mathrm{f}}\hat{T}\hat{G}. \tag{27}$$

Eq. (26) is the mixed (far-near-field) reciprocity relation, when expressed in terms of kernels:

$$\begin{aligned}\left[\overleftrightarrow{S}\left(\mathbf{r}, -\hat{\mathbf{n}}\right)\right]^{\mathrm{T}} &= 4\pi\left[\overleftrightarrow{G}_{\mathrm{Sf}}\left(\hat{\mathbf{n}}, \mathbf{r}\right) - \overleftrightarrow{G}_{\mathrm{f}}\left(\hat{\mathbf{n}}, \mathbf{r}\right)\right] \\ &= 4\pi \lim_{r'\to\infty} r' \exp\left(-ik_1 r'\right)\overleftrightarrow{G}_{\mathrm{S}}\left(r'\hat{\mathbf{n}}, \mathbf{r}\right) \\ &\quad - \exp\left(-ik_1\hat{\mathbf{n}}\cdot\mathbf{r}\right)\left(\overleftrightarrow{I} - \hat{\mathbf{n}}\otimes\hat{\mathbf{n}}\right).\end{aligned} \tag{28}$$

Finally, one may also express $\hat{S}$ as

$$\hat{S} = 4\pi\left(\hat{G}_{\mathrm{Sf}}^{\mathrm{t}} - \hat{G}_{\mathrm{f}}^{\mathrm{t}}\right)\hat{H}. \tag{29}$$

The mixed reciprocity relations can also be proven directly from the underlying scattering problem, see Theorem 6.31 of Ref. [4]

4. Conclusion

The theme of this addendum is aligned with that of the original tutorial [1]. Most importantly, we provided a streamlined derivation of the far-field and mixed reciprocity relations (Eqs. (20), (22) and (28)) through the analysis of the pseudo adjoints of the corresponding linear maps. More generally, the whole operator calculus described in a concise manner can be useful for theoretical analyses of scattering problems for complex scenarios, such as a semi-infinite plane substrate [6], whose effect can be accounted for by a proper modification of the environment operator $\hat{G}$.

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